The Twenty Fifth Katowice–Debrecen Winter Seminar on Functional Equations and Inequalities

Kościelisko-Zakopane, Poland, February 3–6, 2026

The conference is dedicated to the memory of Professor Roman Ger.

List of Participants and talks

Karol Baron, *On some linear functional equations with continuous and bounded solutions*

Zoltán Boros, *Pairs of functions fulfilling an alternative or a cancellative equation on a big set*

Pál Burai, *A negative result concerning an ordering generated by matrix groups*

Katarzyna Chmielewska, -

Jacek Chmieliński, *On approximate preservation of Birkhoff-James orthogonality*

Jacek Chudziak, *On comparability of buying and selling prices*

Borbála Fazekas, *Convergence of sequences of ordered selections*

Attila Gilányi, *On conditional monomial and polynomial functional equations*

Dorota Głazowska, *Complementary means for generalized classical weighted means with respect to classical weighted means*

Eszter Gselmann, *Perturbations of Cauchy differences*

Wojciech Jabłoński, *Weak topologies in rings of formal power series*

Justyna Jarczyk, *Iterates, fractional iterates and flows of Möbius transformations*

Witold Jarczyk, *When are local properties of solutions of iterative functional equations actually global?*

Gergely Kiss, *Questions on the regularity of bisymmetric functions*

Tibor Kiss, *Quasi Graph-Additive Functions – On a Conjecture of Janusz Matkowski*

Radosław Łukasik, *Generalized orthogonality equations in normed spaces*

Gergő Nagy, *Resolving sets in spaces of self-adjoint operators*

Andrzej Olbryś, *Remarks on Sincov's difference*

Zsolt Páles, *On systems of higher-order homogeneous linear differential-algebraic equations*

Paweł Pasteczka, *On the invariance equation for means of generalized power growth*

Patryk Rela, *The robust Orlicz premium principle under uncertainty*

Maciej Sablik, *Roman, my friend and master*

Rafał Stypka, *Approximate Additivity of Operators Approximately Preserving Birkhoff–James Orthogonality*

László Székelyhidi, *Does Spectral Analysis Imply Spectral Synthesis?*

Patricia Szokol, *On some examples and counterexamples about weighted Lagrange interpolation with Exponential- and Hermite weights*

Tomasz Szostok, -

Péter Tóth, *On the equality problem of Bajraktarević means*

Paweł Wójcik, *Norm derivatives and Cauchy-Schwarz Inequality*

Programme

Wednesday

7:30 - 9:00	Breakfast
9:00 - 9:10	**Maciej Sablik**: Opening
	First morning session Chair: **Zsolt Páles**
9:10 - 9:30	**Karol Baron** On some linear functional equations with continuous and bounded solutions
9:35 - 9:55	**Jacek Chudziak** On comparability of buying and selling prices
10:00 - 10:20	**Patryk Rela** The robust Orlicz premium principle under uncertainty
10:25 - 10:45	**Paweł Wójcik** Norm derivatives and Cauchy-Schwarz Inequality
10:50 - 11:15	Coffee break
	Second morning session Chair: **Karol Baron**
11:15 - 11:35	**Dorota Głazowska** Complementary means for generalized classical weighted means with respect to classical weighted means
11:40 - 12:00	**Peter Tóth** On the equality problem of Bajraktarević means
12:05 - 12:25	**Paweł Pasteczka** On the invariance equation for means of generalized power growth
12:30 - 13:00	**Problems and remarks**
13:00 - 14:00	Lunch
	First afternoon session Chair: **László Székelyhidi**
15:00 - 15:20	**Zsolt Páles** On systems of higher-order homogeneous linear differential-algebraic equations
15:25 - 15:45	**Eszter Gselmann** Perturbations of Cauchy differences
15:50 - 16:10	**Gergely Kiss** Questions on the regularity of bisymmetric functions
16:15 - 16:45	Coffee break
	Second afternoon session Chair: **Maciej Sablik**
16:45 - 17:05	**Justyna Jarczyk** Iterates, fractional iterates and flows of Möbius transformations
17:10 - 17:30	**Witold Jarczyk** When are local properties of solutions of iterative functional equations actually global?
17:35 - 17:55	**Tibor Kiss** Quasi Graph-Additive Functions – On a Conjecture of Janusz Matkowski
18:00 - 19:00	Dinner

Thursday

7:30 - 9:00	Breakfast
	First morning session Chair: **Attila Gilányi**
9:00 - 9:20	**László Székelyhidi** Does Spectral Analysis Imply Spectral Synthesis?
9:25 - 9:45	**Patricia Szokol** On some examples and counterexamples about weighted Lagrange interpolation with Exponential- and Hermite weights
9:50 - 10:10	**Borbála Fazekas** Convergence of sequences of ordered selections
10:15 - 10:35	**Pál Burai** A negative result concerning an ordering generated by matrix groups
10:40 - 11:15	Coffee break
	Second morning session Chair: **Witold Jarczyk**
11:15 - 11:35	**Zoltán Boros** Pairs of functions fulfilling an alternative or a cancellative equation on a big set
11:40 - 12:00	**Wojciech Jabłoński** Weak topologies in rings of formal power series
12:05 - 12:25	**Andrzej Olbryś** Remarks on Sincov's difference
12:30 - 13:00	**Problems and remarks**
13:00 - 14:00	Lunch
	FREE TIME
	For the participants staying in the hotel:
16:15 - 16:45	Coffee break
19:00	Festive dinner

Friday

7:30 - 9:00	Breakfast
	First morning session Chair: **Gergely Kiss**
9:00 - 9:20	**Jacek Chmieliński** On approximate preservation of Birkhoff-James orthogonality
9:25 - 9:45	**Rafał Stypka** Approximate Additivity of Operators Approximately Preserving Birkhoff–James Orthogonality
9:50 - 10:10	**Radosław Łukasik** Generalized orthogonality equations in normed spaces
10:15 - 10:35	**Gergő Nagy** Resolving sets in spaces of self-adjoint operators
10:40 - 11:20	Coffee break
	Second morning session Chair: **Jacek Chmieliński**
11:20 - 11:40	**Attila Gilányi** On conditional monomial and polynomial functional equations
11:45 - 12:05	**Maciej Sablik** Roman, my friend and master
12:10 - 12:25	**Problems and remarks**
12:25 - 12:30	**Zsolt Páles**: Closing
12:30 - 13:30	Lunch

Abstracts

The 25th Katowice–Debrecen Winter Seminar
on Functional Equations and Inequalities
Kościelisko-Zakopane, Poland, February 3–6, 2026

On some linear functional equations with continuous and bounded solutions

Karol Baron
University of Silesia

The research of Witold Jarczyk, initiated in [3] and continued in [2] and [4], showed that sets of first order non-homogeneous linear functional equations having a continuous solution are small from the topological point of view. We return to these qualitative considerations with higher orders equations in mind.

In the talk we will briefly describe our approach to the equation

$$\varphi(x) = \int_\Omega \varphi\big(f(x,\omega)\big)\mathbb{P}(d\omega) + F(x)$$

assuming that $(\Omega, \mathcal{A}, \mathbb{P})$ is a probability space, (X, ρ) is a complete and separable metric space, $f : X \times \Omega \to X$ is continuous with respect to the first variable and measurable for $\mathcal{A}$ with respect to the second variable, and

$$\int_\Omega \rho\big(f(x,\omega), f(z,\omega)\big)\mathbb{P}(d\omega) \le \beta\big(\rho(x,z)\big) \quad \text{for } x, z \in X$$

with a concave $\beta : [0,\infty) \to [0,\infty)$ such that $\beta(t) < t$ for $t \in (0,\infty)$.

Based on the ideas from [1] and slightly strengthening the assumption made there about the function f, we now consider solutions defined on complete and separable metric spaces, and not, as in [1], on compact spaces.

References

[1] Karol Baron, *On some linear functional equations with continuous solutions*, Aequationes Math. **99** (2025), 2689–2698.
[2] Witold Jarczyk, *A category theorem for linear functional equations in the indeterminate case*, Bull. Acad. Polon. Sci. Sér. Sci. Math. **29** (1981), 371–372.
[3] Witold Jarczyk, *On a set of functional equations having continuous solutions*, Glasnik Mat. **17(37)** (1982), 59–64.
[4] Witold Jarczyk, *On linear functional equations in the determinate case*, Glasnik Mat. **18(38)** (1983), 91–102.

Pairs of functions fulfilling an alternative or a cancellative equation on a big set

Zoltán Boros
University of Debrecen

(joint work with Rayene Menzer)

In our investigations on the alternative equation

$$f(x)g(y) = 0\,, \tag{1}$$

we have established the following result. Let $k\,,\, m \in \mathbb{N}$. Suppose that $f : \mathbb{R}^k \to \mathbb{C}$ and $g : \mathbb{R}^m \to \mathbb{C}$ satisfy (1) for all $(x, y) \in D$, where $D \subseteq \mathbb{R}^{k+m}$ has a positive $k + m$ dimensional Lebesgue measure or D is a second category Baire set. Then f or g vanishes on a set of positive (k or m dimensional) Lebesgue measure or on a second category Baire set, respectively.

Motivated by a question of Peter Eliaš, investigations were extended the functional equation

$$F(f(x), g(y)) = 0\,, \tag{2}$$

where F is a given operation. Assuming that $S \subseteq \mathbb{C} \times \mathbb{C}$ and $F : S \to \mathbb{C}$ is locally cancellative in the sense that, for all $z \in \mathbb{C}$ and $w \in \mathbb{C}$, the equation $F(z, w) = 0$ has at most one solution in its first (resp., second) variable if the second (resp., first) variable is fixed, we can establish the following theorem: If $f : \mathbb{R}^k \to \mathbb{C}$ and $g : \mathbb{R}^m \to \mathbb{C}$ satisfy (2) for all $(x, y) \in D$, where $D \subseteq \mathbb{R}^{k+m}$ has a positive $k + m$ dimensional Lebesgue measure or D is a second category Baire set, then f and g are constants on sets of positive (k and m dimensional) Lebesgue measure or on second category Baire sets, respectively.

A negative result concerning an ordering generated by matrix groups

Pál Burai
Budapest University of Technology and Economics
(joint work with Hanna Koller)

Let $G \subset GL_n(\mathbb{R})$ be a matrix group, where $GL_n(\mathbb{R})$ denotes the general linear group of degree n.

An $x \in \mathbb{R}^n$ is majorized by a $y \in \mathbb{R}^n$ with respect to the group G, if

$$x \in conv(O_G(y)),$$

where $O_G(y)$ is the orbit of y under G and *conv* denotes the convex hull operator.

In this talk we present a negative result concerning the characterization of ordering generated by cyclic, doubly stochastic matrices. Possible further research directions are also mentioned.

On approximate preservation of Birkhoff-James orthogonality

Jacek Chmieliński
University of the National Education Commission, Krakow
(joint work with Rafał Stypka)

We consider additive operators that approximately preserve (or reverse) the Birkhoff-James orthogonality relation. In particular, we show that such operators must be linear.

References

[1] J. Chmieliński, R. Stypka, *Additive operators approximately preserving Birkhoff–James orthogonality*, Aequationes Math. **99** (2025), no. 6, 2847–2854.

On comparability of buying and selling prices

Jacek Chudziak

University of Rzeszów

The buying price is a real number that expresses the maximal amount at which the investor is willing to buy a risky asset X. If the investor has already received X, then the selling price expresses the minimal price at which he is willing to sell X. In the expected utility model, where the risks are represented by essentially bounded random variables on a given probability space, the buying price $B_u(w, X)$ for X at a wealth level w is defined implicitly through the equation

$$E[u(w + X - B_u(w, X))] = u(w),$$

where $u : \mathbb{R} \to \mathbb{R}$ is a continuous and strictly increasing function. The selling price $S_u(w, X)$ for X at a wealth level w is given by

$$E[u(w + X)] = u(w + S_u(w, X)).$$

Several properties of the buying and selling prices under the expected utility model have been investigated in [1]. In the talk, we introduce the buying and selling prices under rank-dependent utility and we present some results concerning the comparability problem for such prices.

Reference

[1] Michał Lewandowski, *Buying and selling price for risky lotteries and expected utility theory with gambling wealth*, J. Risk Uncertainty **48** (2014), 253–283.

Convergence of sequences of ordered selections

Borbála Fazekas

University of Debrecen

(joint work with István Fazekas)

Our results are generalizations of well-known theorems on convergence of permutation sequences to permutons. We introduce a convergence notion for ordered selections, which is based on subpermutation densities and convergences of the marginal distributions. We also introduce a family of probability measures called generalized permutons and we embed the set of ordered selections to the set of generalized permutons. We prove that any convergent sequence of ordered selections has a limit which is a generalized permuton. Moreover, any generalized permuton is the limit of a sequence of ordered selections.

Reference

[1] Fazekas, I., Fazekas, B., *Convergence of sequences of ordered selections*, preprint.

On conditional monomial and polynomial functional equations

Attila Gilányi
University of Debrecen

Dedicated to the memory of Roman Ger

Problems related to extensions and conditionally fulfilled properties often occur in the field of functional equations and their applications (cf., e.g., the 'Program' [1] announced by János Aczél). Related to this topic, some results for conditional monomial and polynomial functional equations are presented.

Reference

[1] J. Aczél, *5. Remark*, Report of meeting, Aequationes Math. **69** (2005), 183.

Complementary means for generalized classical weighted means with respect to classical weighted means

Dorota Głazowska
University of Zielona Góra

Let $\mathcal{A}_\lambda$, $\mathcal{G}_\lambda$ and $\mathcal{H}_\lambda$, denote, respectively, the bivariable classical weighted arithmetic, geometric and harmonic means with weight $\lambda \in (0,1)$.

Under some simple conditions on the real functions f, g and h, defined on an interval $I \subset \mathbb{R}$ or $I \subset (0,\infty)$, the bivariable functions A_f, G_g and H_h, given respectively by

$$A_f(x,y) = f(x) + y - f(y), \quad G_g(x,y) = \frac{g(x)}{g(y)}y, \quad H_h(x,y) = \frac{xy}{x - h(x) + h(y)},$$

are means on the interval I. These means are natural generalization, respectively, of the classical weighted arithmetic, geometric and harmonic means.

Fixing arbitrarily $\lambda \in (0,1)$ and choosing for $K\colon I^2 \to I$ one of these three classical weighted means, for arbitrary mean $M\colon I^2 \to I$, we examine when the function $N\colon I^2 \to \mathbb{R}$ satisfying the equality

$$K \circ (M,N) = K$$

is a mean, that is when the mean K is invariant with respect to the mean type-mapping $(M,N) : I^2 \to I^2$, where $I \subset \mathbb{R}$ or $I \subset (0,+\infty)$ is an interval.

Recall that if M, N are continuous and strict means, then there exists a unique (M,N)-invariant mean. In the case when K is the unique (M,N)-invariant mean, we say that N is a complementary mean to M with respect to K.

The obtained results are applied to determine complementary means for generalized classical weighted means with respect to classical weighted means.

This is a report on research made jointly with Janusz Matkowski

Perturbations of Cauchy differences

Eszter Gselmann
University of Debrecen

(joint work with Tomasz Małolepszy and Janusz Matkowski)

In this talk we investigate functional equations arising from perturbations of Cauchy differences. We study equations of the form

$$f(x+y) - f(x) - f(y) = B(x,y) \quad \text{or} \quad f(xy) - f(x)f(y) = B(x,y)$$

where B is a biadditive mapping, and also more general cases where the inhomogeneity depends on unknown functions

$$f(x+y) - f(x) - f(y) = \alpha xy$$

$$f(x+y) - f(x) - f(y) = \alpha(xy)$$

$$f(x+y) - f(x) - f(y) = \alpha(x)(y).$$

Our results extend previous works on the bilinearity of the Cauchy exponential difference by Horst Alzer and Janusz Matkowski. We characterize solutions under various structural and regularity assumptions, including additive and exponential Cauchy differences, and show that solutions often reduce to additive functions, exponential polynomials, or combinations thereof. For Levi-Civita type equations, we provide explicit representations of solutions in terms of exponential polynomials. Furthermore, we determine conditions under which real-valued solutions exist and describe their forms. The talk concludes with open problems concerning equations of similar type that cannot be solved by the methods presented here, suggesting directions for future research.

Weak topologies in rings of formal power series

Wojciech Jabłoński
Jan Kochanowski University of Kielce

The operation of substitution (composition) for formal power series is easily defined in a maximal ideal of the ring of formal power series. A definition of the substitution in whole ring of formal power series was given without any explanation in [1,2,3,4]. In order to show the sense of this definition we introduce topologies which are essentially weaker than the strong topology defined in purely algebraic way in the ring of formal power series. As applications we obtain properties of the Hasse derivative and an elementary proof of the existence of composition in a ring of formal power series of several variables over a topological ring.

References

[1] D. Bugajewski, A. Galimberti, P. Maćkowiak, *On composition and Right Distributive Law for formal power series of multiple variables*, Aequat. Math. 99 (2025), 21 – 35.
[2] X.-X. Gan, *A generalized chain rule for formal power series*, Comm. Math. Anal. 2 (2007), 37-44.
[3] X.-X. Gan, D. Bugajewski, *A note on formal power series*, Comment. Math. Univ. Carolin. 51 (2010), 595–604.
[4] X.-X. Gan, N. Knox, *On composition of formal power series*, Int. J. Math. Math. Sci. 30 (2002), no. 12, 761–770.

Iterates, fractional iterates and flows of Möbius transformations

Justyna Jarczyk
University of Zielona Góra

(joint work with Steven Finch and Witold Jarczyk)

Let $f : \mathbb{C} \cup \{\infty\} \to \mathbb{C} \cup \{\infty\}$ be any Möbius transformation given by

$$f(z) = \begin{cases} \frac{az+b}{cz+d}, & \text{when} \quad z \in \mathbb{C} \setminus \left\{-\frac{d}{c}\right\}, \\ \infty, & \text{when} \quad z = -\frac{d}{c}, \\ \frac{a}{c}, & \text{when} \quad z = \infty, \end{cases}$$

where $a, b, c, d \in \mathbb{C}$ are such that $c \neq 0$ and $ad - bc \neq 0$. Using a recurrent method we find a simple explicit form of iterates of f. Then, in a neighbourhood of the attractive fixed point ξ of f, we determine effective formulas for solutions σ of the Schröder equation

$$\sigma(f(z)) = f'(\xi)\sigma(z)$$

which are differentiable at ξ. Using this we can find the form of fractional iterates (i.e. iterative roots) of f and, in some cases, embed f into a flow.

When are local properties of solutions of iterative functional equations actually global?

Witold Jarczyk
University of Zielona Góra

(joint work with Paweł Pasteczka)

The extendability of properties of solutions of simultaneous iterative equations is considered. It turns out that, under some additional assumptions, if the solution of

$$\varphi(f_t(x)) = g(x, \varphi(x)), \quad t \in T,$$

exists, then it is not only unique but its local properties are actually global. More precisely, in the case when the indicated function solves the simultaneous equations, each its local property can be propagated to the entire domain.

Questions on the regularity of bisymmetric functions

Gergely Kiss
Corvinus University of Budapest and HUN-REN Alfréd Rényi Institute of Mathematics

In recent years, there has been renewed interest in the role of regularity assumptions in Aczél's characterization theorem for (weighted) quasi-arithmetic means, with particular emphasis on how (and to what extent) continuity can be eliminated. Closely related work investigates the regularity of monotone bisymmetric functions defined on real intervals, aiming to understand which structural conclusions can be derived from purely algebraic identities together with monotonicity.

These developments also leave a number of questions open. In this talk, I will briefly survey several recent results on the topic and then turn to questions that, in my view, merit broader attention. Starting from the two-variable setting, I will formulate and discuss concrete questions. I will also present multivariable analogues and pose further open problems.

Most of the results to be discussed are joint work with Pál Burai and Patrícia Szokol.

REFERENCES

[1] János Aczél, *Functional equations in several variables*, volume 31 of *Encyclopedia of Mathematics and its Applications*, Cambridge University Press, Cambridge, 1989.

[2] Pál Burai, Gergely Kiss and Patrícia Szokol, *Characterization of quasi-arithmetic means without regularity condition*, Acta Math. Hung. **69** (1962), 769–772.

[3] Pál Burai, Gergely Kiss and Patrícia Szokol, *A dichotomy result for strictly increasing bisymmetric maps*, Acta Math. Hung. **165** (2021), 474–485.

[4] Gergely Kiss, *On noncontinuous bisymmetric strictly monotone operations*, submitted, 2025.

Quasi Graph-Additive Functions – On a Conjecture of Janusz Matkowski

Tibor Kiss
University of Debrecen

On September 18, 2025, the fourth day of the *21st International Conference on Functional Equations and Inequalities*, D. Głasowska discussed in her talk a conjecture originally formulated by J. Matkowski [5], concerning the continuous solutions of the functional equation

$$f\big(f(-x)+x\big) = f\big(-f(x)\big) + f(x), \qquad x \in \mathbb{R}.$$

Motivated by W. Jarczyk's related results [2, 3], Matkowski conjectured that, loosely speaking, the continuous solutions of the above equation are precisely those that are positively homogeneous on the non-positive and on the non-negative half-line, respectively. Although the conjecture was well-founded, it turned out that the family of continuous solutions is much richer.

The talk will focus on continuous solutions that are positively homogeneous on the non-positive half-line. Furthermore, we provide a sufficient condition under which the statement appearing in Matkowski's conjecture holds.

References

[1] Głazowska, D., Matkowski, J., *Weakly associative functions*, Aequat. Math. 99, 1827-1841 (2025).

[2] Jarczyk, W., *On continuous functions which are additive on their graphs*, Berichte Math.-Statist. Section in der Forschungsgesellschaft Joanneum - Graz 292 (1988).

[3] Jarczyk, W., *A recurrent method of solving iterative functional equations*, Prace Matematyczne Uniw. Si. w Katowicach 1206 (1991).

[4] Kiss, T., *A Counterexample to Matkowski's Conjecture for Quasi Graph-Additive Functions*, submitted (2026).

[5] Matkowski, J., *Weakly associative functions and means - new examples and open questions*, Aequat. Math. 99, 2581-2597 (2025).

Generalized orthogonality equations in normed spaces

Radosław Łukasik
University of Silesia

(joint work with Abayomi Epebinu)

Let X, Y be real normed spaces and let ρ'_+, ρ'_- be norm derivatives. In this talk, we solve a system of functional equations

$$\begin{cases} \rho'_+(f(x), f(y)) = g(x)\rho'_+(x, y) \\ \rho'_-(f(x), f(y)) = g(x)\rho'_-(x, y) \end{cases}$$

where the functions $f\colon X \to Y$ and $g\colon X \to \mathbb{R}$ are unknown.

REFERENCE

[1] A. Epebinu, R. Łukasik, Generalized orthogonality equations in normed spaces, Aequat. Math. 100:5 (2026).

Resolving sets in spaces of self-adjoint operators

Gergő Nagy
University of Debrecen

Concerning certain investigations related to metric spaces, in particular the study of special isometries, it is useful to find such sets which generate these spaces in a kind of metric sense. The resolving sets are particular examples of collections with this property, they are those subsets of a metric space X for which any element of X can be uniquely identified by its distances to the points of such a subset.

This talk is devoted to results on resolving sets in particular metric spaces of self-adjoint operators. These spaces include that of the density operators on a complex Hilbert space H, i.e., the positive trace class operators on H with trace 1. Further, the metrics considered come from the Schatten norms. We present results stating that the extreme points of the convex subset of all density operators on H form a resolving set of the latter collection. These statements tell us that the set of extreme points of a certain convex subset C in a normed space generates C in a kind of metric sense. This result is a counterpart of a corollary of the famous Krein-Milman theorem stating that the set E of extreme points of a convex compact set K in a locally convex topological vector space generates K in a certain linear algebraic-topological sense, namely the closed convex hull of E is K.

Remarks on Sincov's difference

Andrzej Olbryś
University of Silesia

Answering the question given by Ludwig Reich we characterize a three place map which can be expressed as the Sincov difference i.e. a map of the form

$$F(x,z) - F(x,y) - F(y,z),$$

as well as, its pexiderized version

$$F(x,z) - G(x,y) - H(y,z).$$

REFERENCES

[1] A.R.Baias, D.Otrocol and M.Rus, *Report of Meeting The* 61^{st} *International Symposium on Functional Equations, Cluj-Napoca (Romania), June 15-21, 2025*, Aequat. Math. **99** (2025), 2457-2479, https://doi.org/10.1007/s00010-025-01236-8.

On systems of higher-order homogeneous linear differential-algebraic equations

Zsolt Páles
University of Debrecen
(joint work with Eszter Gselmann)

In the talk, we consider the following system of homogeneous linear differential-algebraic equations

$$A_k y^{(k)}(t) + \cdots + A_1 y'(t) + A_0 y(t) = 0, \qquad t \in I,$$

where $k, n, m \in \mathbb{N}$, $A_k, \ldots, A_1, A_0 \in \mathbb{C}^{m\times n}$, I is a nonempty open subinterval of $\mathbb{R}$, and $y\colon I \to \mathbb{C}^n$ denotes a sufficiently smooth unknown function.

The above sytem is called *nondegenerate* if, for some number $\lambda \in \mathbb{C}$, the rank of the $m \times n$ matrix

$$\lambda^k A_k + \cdots + \lambda A_1 + A_0$$

equals n. Our main result shows that the solution space of the above system of differential-algebraic equations is finite-dimensional if and only if this nondegeneracy condition holds. Furthermore, in this case, the solution space is a finite-dimensional subspace of the space of $\mathbb{C}^n$-valued exponential polynomials.

On the invariance equation for means of generalized power growth

Paweł Pasteczka
University of Rzeszów

We generalize the result of (Witkowski, 2014) which binds orders of homogeneous, symmetric means $M, N, K\colon \mathbb{R}_+^2 \to \mathbb{R}$ of power growth that satisfy the invariance equation

$$K(M(x,y), N(x,y)) = K(x,y)$$

to the broader class of means. Moreover, we define the lower- and the upper-order which gives us insight into the order of the solution of this equation in the case when means do not belong to this class.

References

[1] Paweł Pasteczka, *On the invariance equation for means of generalized power growth*, Math. Inequal. Appl. **27** (2024), 691–702.

The robust Orlicz premium principle under uncertainty

Patryk Rela
University of Rzeszów
(joint work with Jacek Chudziak)

Assume that $(\Omega, \mathcal{F})$ is a measurable space and $\mu_0 : \mathcal{F} \to [0,1]$ is a capacity, that is a monotone set function satisfying $\mu_0(\emptyset) = 0$ and $\mu_0(\Omega) = 1$. For a capacity μ_0, by S_{μ_0} we denote a family of all capacities $\mu : \mathcal{F} \to [0,1]$ satisfying, for every $A \in \mathcal{F}$, the following condition

$$\mu_0(A) = 0 \implies \mu(A) = 0. \tag{1}$$

Let $\mathcal{X}_{\mu_0}$ be a family of all non-negative $\mathcal{F}$-measurable functions $X : \Omega \to [0, \infty)$ such that $\mu_0(\{X > t\}) = 0$ for some $t \in \mathbb{R}$. The robust Orlicz premium principle under uncertainty for the risk $X \in \mathcal{X}_{\mu_0}$ is defined through the equation

$$H_{\alpha,S,\Phi}(X) := \inf \left\{ k > 0 : \sup_{\mu \in S} E_\mu \left[\Phi\left(\frac{X}{k}\right)\right] \leq 1 - \alpha \right\}, \tag{2}$$

where $\alpha \in [0,1)$ is a given parameter, $S \subset S_{\mu_0}$ and the function $\Phi : [0,\infty) \to [0,\infty)$ is a normalized Young function, that is a strictly increasing and convex function, satisfying $\Phi(0) = 0$ and $\Phi(1) = 1$. Moreover

$$E_\mu [X] = \int\limits_0^\infty \mu(\{X > x\})dx \quad \text{for} \quad X \in \mathcal{X}_{\mu_0},$$

is the Choquet integral with respect to the capacity μ.

The aim of this talk is to prove the existence of the robust Orlicz premium defined by (2) and to characterize its several important properties.

REFERENCES

[1] Fabio Bellini, Roger J.A. Laeven, Emanuela Rosazza Gianin *Robust return risk measures*, Mathematics and Financial Economics **12** (2018), 5-32.

Roman, my friend and master

Maciej Sablik
University of Silesia

These are few words (and some pictures) illustrating my half a century long friendship with the late Roman Ger.

Approximate Additivity of Operators Approximately Preserving Birkhoff–James Orthogonality

Rafał Stypka
University of the National Education Commission, Krakow

We consider approximately additive operators between real normed spaces that approximately preserve Birkhoff–James orthogonality. Let X be a normed space over the field $\mathbb{K} \in \{\mathbb{R}, \mathbb{C}\}$ with $\dim X \geq 2$, and let $\varepsilon \in [0, 1)$. We introduce the following notions.

- *Birkhoff–James orthogonality*:
$$x \perp_B y \iff \|x + \lambda y\| \geq \|x\| \quad \text{for every } \lambda \in \mathbb{K}.$$
- *Approximate Birkhoff–James orthogonality*:
$$x \underset{\varepsilon}{\perp}_B y \iff \|x + \lambda y\| \geq (1 - \varepsilon)\|x\| \quad \text{for every } \lambda \in \mathbb{K}.$$

An operator $T \colon X \to Y$ between normed spaces is said to be an *approximately Birkhoff–James orthogonality preserving operator* if
$$x \perp_B y \quad \Rightarrow \quad T(x) \underset{\varepsilon}{\perp}_B T(y) \quad \text{for all } x, y \in X.$$

The main result of the paper in the setting of real normed spaces establishes that, in a neighborhood of an approximately additive operator, there exists a linear, bounded, and continuous operator which approximately preserves Birkhoff–James orthogonality. Our approach relies on the geometric structure of normed spaces and on inequalities derived from the definition of approximate Birkhoff–James orthogonality.

The obtained result constitutes a generalization of the theorems proved in [1], where it was shown that every additive operator that approximately preserves Birkhoff–James orthogonality is necessarily linear.

References

[1] J. Chmieliński, R. Stypka, *Additive operators approximately preserving Birkhoff-James orthogonality*, Aequat. Math. **99** (2025), 2847–2854.

Does Spectral Analysis Imply Spectral Synthesis?

László Székelyhidi
University of Debrecen

Given a system of convolution type functional equations spectral analysis means that every nontrivial extension of the system has exponential solutions. The system is called trivial, if the only solution is the zero function. In terms of linear algebra this means that every nonzero invariant subspace of the solution space includes eigenvectors. A seemingly stronger property of a system is spectral synthesis: it means that in every nontrivial extension of the system the exponential monomial solutions span a dense subspace. In the finite dimensional case this means that the generalized eigenvectors form a basis in every invariant subspace of the solution space. In this talk we show that, in fact, the spectral analysis property implies the spectral synthesizability unless the underlying group has infinitely many linearly independent additive functions.

On some examples and counterexamples about weighted Lagrange interpolation with Exponential- and Hermite weights

Patricia Szokol

University of Debrecen

(joint work with Szilárd Révész)

The famous Bernstein conjecture about optimal node systems of classical polynomial Lagrange interpolation, standing unresolved for over half a century, was solved by T. Kilgore in 1978 [1]. Immediately following him, also the additional conjecture of Erdős was solved by deBoor and Pinkus [2]. These breakthrough achievements were built on a fundamental auxiliary result on nonsingularity of derivative (Jacobian) matrices of certain interval maxima in function of the nodes. After the above breakthrough, a considerable effort was made to extend the results to the case of at least certain restricted classes of functions and Chebyshev-Haar subspaces.

Our aim is to analyze whether this key nonsingularity property holds for exponentially weighted interpolation on a half-line, as well as under Hermite weights on the entire real line. Our case studies also present counterexamples that demonstrate the singularity of the derivative matrices, thereby calling into question the validity of the Bernstein and Erdős conjectures.

References

[1] Theodore Kilgore, *A characterization of the Lagrange interpolating projection with minimal Tchebycheff norm*, J. Approx. Th. **24** (1978), 273–288.

[2] Carl de Boor, Allan Pinkus, *Proof of the conjectures of Bernstein and Erdős concerning the optimal nodes for polynomial interpolation*, J. Approx. Th. **24** (1978), 289–303.

On the equality problem of Bajraktarević means

Péter Tóth
University of Debrecen

We discuss the equality problem of two variable generalized Bajraktarević means. That is, we investigate the functional equation

$$f^{(-1)}\left(\frac{p_1(x)f(x)+p_2(y)f(y)}{p_1(x)+p_2(y)}\right)=g^{(-1)}\left(\frac{q_1(x)g(x)+q_2(y)g(y)}{q_1(x)+q_2(y)}\right)\qquad (x,y\in I)$$

where I is an open interval, $f,g:I\to\mathbb{R}$ are strictly monotone functions, $f^{(-1)}$ and $g^{(-1)}$ are their generalized left inverses, respectively, while $p_1,p_2,q_1,q_2:I\to\mathbb{R}_+$ are positive weight functions. The strongest results about the symmetric version (i.e. the case when $p_1=p_2$ and $q_1=q_2$) and non-symmetric version of this problem are contained in the papers [1] and [2], respectively.

In those works continuous differentiability is assumed for the generator functions f,g and one of the weight functions p_i as well. In our talk we eliminate some of these regularity assumptions, and rely only on the differentiability of some weight functions.

References

[1] Zs. Páles and A. Zakaria, *On the equality problem of two-variable Bajraktarević means under first-order differentiability assumptions*, Aequat. Math. **97** (2023), 279–294.

[2] Zs. Páles and A. Zakaria, *On the equality of generalized Bajraktarević means under first-order differentiability assumptions*, Aequat. Math. **99** (2025), 2485–2503.

Norm derivatives and Cauchy-Schwarz Inequality

Paweł Wójcik
University of the National Education Commission, Krakow
(joint work with S.M. Enderami, M. Abtahi and A . Zamani)

Let X be a complex normed space. Based on the norm derivative $\rho_+\colon X \times X \to \mathbb{R}$, we define a mapping $\rho_\infty\colon X \times X \to \mathbb{C}$ by the following formula:

$$\rho_\infty(x, y) := \frac{1}{\pi} \int_0^{2\pi} e^{i\theta} \rho_+(x, e^{i\theta} y) d\theta.$$

The mapping ρ_∞ has a good response to some geometrical properties of X. Some problem connected with the Cauchy-Schwarz Inequality will be discussed.

References

[1] S.M. Enderami, M. Abtahi, A. Zamani, P. Wójcik, *An orthogonality relation in complex normed spaces based on norm derivatives*, Linear Multilinear Algebra **72**(4) (2024), 687–705.